Calculus 1 Name____________________
Assessment #3 Review Date____________________

1) Use the limit definition of a derivative to find the derivative of $f(x)=2x^2+5$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\lim_{h\to 0}\frac{2(x+h)^2+5-(2x^2+5)}{h}$$
$$\lim_{h\to 0}\frac{2(x^2+2xh+h^2)+5-2x^2-5}{h}$$
$$\lim_{h\to 0}\frac{2x^2+4xh+2h^2+5-2x^2-5}{h}$$
$$\lim_{h\to 0}\frac{4xh+2h^2}{h}$$
$$\lim_{h\to 0}4x+2h$$
$$f'(x)=4x$$

2) Use the limit definition of a derivative to find the derivative of $f(x)=4x^2+3x-7$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$\lim_{h\to 0}\frac{4(x+h)^2+3(x+h)-7-(4x^2+3x-7)}{h}$$
$$\lim_{h\to 0}\frac{4(x^2+2xh+h^2)+3x+3h-7-4x^2-3x+7}{h}$$
$$\lim_{h\to 0}\frac{4x^2+8xh+4h^2+3x+3h-7-4x^2-3x+7}{h}$$
$$\lim_{h\to 0}\frac{8xh+4h^2+3h}{h}$$
$$\lim_{h\to 0}8x+4h+3$$
$$f'(x)=8x+3$$

3) $f(x)=7x^4-5x^3+6x-3$

$$f'(x)=28x^3-15x^2+6$$

5) $g(x)=\dfrac{11}{\sqrt[6]{x^5}}-\dfrac{3\sqrt{x}}{5\sqrt[3]{x^2}}$

$$g(x)=11x^{-5/6}-\frac{3}{5}x^{1/2}x^{-2/3}$$
$$g(x)=11x^{-5/6}-\frac{3}{5}x^{-1/6}$$
$$g'(x)=\frac{-55}{6}x^{-11/6}+\frac{3}{30}x^{-7/6}$$
$$g'(x)=\frac{-55}{6}x^{-11/6}+\frac{1}{10}x^{-7/6}$$

4) $V=\dfrac{3}{4}\pi r^2+3r-4$

$$V'=\frac{6}{4}\pi r+3$$
$$V'=\frac{3}{2}\pi r+3$$

6) $y=\sin^6 x-\tan\left(5x^3+4x\right)$

$$y'=(6\sin^5 x)(\cos x)-\sec^2(5x^3+4x)(15x^2+4)$$

7) $h(x) = \cot(2x^3 - x + 4)$

$h'(x) = -\csc^2(2x^3 - x + 4)(6x^2 - 1)$

8) $f(x) = (4x - 7)^3(3x^2 + x)^5$

$f'(x) = (4x - 7)^3 5(3x^2 + x)^4(6x + 1) + 3(4x - 7)^2(4)(3x^2 + x)^5$

9) $f(x) = \dfrac{\csc x}{6x^2 + x}$

$$f'(x) = \frac{(6x^2 + x)(-\csc x \cot x) - \csc x(12x + 1)}{(6x^2 + x)^2}$$

Or

$f(x) = \csc x\left(6x^2 + x\right)^{-1}$

$f'(x) = -\csc x \cot x\left(6x^2 + x\right)^{-1} + \csc x\left(-1\left(6x^2 + x\right)^{-2}\left(12x + 1\right)\right)$

10) $k(x) = \dfrac{6\tan x}{\left(x^3 - 7x^2 + 2x\right)^4}$

$$k'(x) = \frac{(x^3 - 7x^2 + 2x)^4(6\sec^2 x) - 6\tan x(4(x^3 - 7x^2 + 2x)^3(3x^2 - 14x + 2)}{(x^3 - 7x^2 + 2x)^8}$$

$$k'(x) = \frac{(x^3 - 7x^2 + 2x)(6\sec^2 x) - 6\tan x(4)(3x^2 - 14x + 2)}{(x^3 - 7x^2 + 2x)^5}$$

Or

$k(x) = 6\tan x\left(x^3 - 7x^2 + 2x\right)^{-4}$

$k'(x) = 6\sec^2 x\left(x^3 - 7x^2 + 2x\right)^{-4} + 6\tan x\left(-4\left(x^3 - 7x^2 + 2x\right)^{-5}\left(3x^2 - 14x + 2\right)\right)$

11) $y = \dfrac{\left(2x^2 + 6\right)^4}{\left(3x^3 - x\right)^5}$

$$y' = \frac{(3x^3 - x)^5 4(2x^2 + 6)^3(4x) - (2x^2 + 6)^4 5(3x^3 - x)^4(9x^2 - 1)}{(3x^3 - x)^{10}}$$

$$y' = \frac{(3x^3 - x)4(2x^2 + 6)^3(4x) - (2x^2 + 6)^4 5(9x^2 - 1)}{(3x^3 - x)^6}$$

12) $g(x)=\dfrac{2}{\sin^2 x}$

$$y'=\frac{\sin^2 x(0)-2(2\sin x)(\cos x)}{\sin^4 x}$$
$$y'=\frac{-4\sin x\cos x}{\sin^4 x}$$
$$y'=\frac{-4\cos x}{\sin^3 x}$$

13) Find the slope of the tangent line of $f(x)=\cos(5x^4+6x^2-2x)$ at $x=\dfrac{2\pi}{3}$.

$$f'(x)=-\sin(5x^4+6x^2-2x)(20x^3+12x-2)$$
$$f'\left(\frac{2\pi}{3}\right)=178.811$$

14) Find the slope of the tangent line of $f(x)=(2x^2-3)(5x^3+4x)$ at $x=1.54$.

2 Methods

Foil first then take the derv.

$$f(x)=10x^5-15x^3+8x^3-12x$$
$$f(x)=10x^5-7x^3-12x$$
$$f'(x)=50x^4-21x^2-12$$
$$f'(1.54)=219.421$$

Use Product Rule to take the derivative

$$f'(x)=(2x^2-3)(15x^2+4)+(4x)(5x^3+4x)$$
$$f'(1.54)=219.421$$

15) Find the equation of the tangent line of $y=5\sin^4 x$ at $x=\dfrac{3\pi}{8}$.

$$y'=20\sin^3 x\cos x$$

Plug in $x=\dfrac{3\pi}{8}$ to the original problem to find y. → y = 3.643

Plug in $x=\dfrac{3\pi}{8}$ to the derivative to find the m. → m = 6.036

$$3.643=6.036\left(\frac{3\pi}{8}\right)+b$$
$$-3.468=b$$
$$y=6.036x-3.468$$

16) Find the second derivative of $g(x)=\sqrt[4]{x^5}-\dfrac{7}{5x^4}+5x$.

$$g(x)=x^{5/4}-\frac{7}{5}x^{-4}+5x$$
$$g'(x)=\frac{5}{4}x^{1/4}+\frac{28}{5}x^{-5}+5$$
$$g''(x)=\frac{5}{16}x^{-3/4}-28x^{-6}$$

17) Find the second derivative of $f(x)=\frac{3x-4}{7x^2+2}$.

$$f'(x)=\frac{(7x^2+2)(3)-(3x-4)(14x)}{(7x^2+2)^2}$$

$$f'(x)=\frac{21x^2+6-42x^2+56x}{(7x^2+2)^2}$$

$$f'(x)=\frac{-21x^2+56x+6}{(7x^2+2)^2}$$

$$f''(x)=\frac{(7x^2+2)^2(-42x+56)-(-21x^2+56x+6)2(7x^2+2)(14x)}{(7x^2+2)^4}$$

$$f''(x)=\frac{(7x^2+2)(-42x+56)-(-21x^2+56x+6)2(14x)}{(7x^2+2)^3}$$